Open Door Education's

# SAT Math Guide

# Table of Contents

## The Structure of the Math Sections

- The Math sections are the last two sections of the SAT.
- Both Math sections give us 35 minutes for 22 questions.
- Section 4 is adaptive and may be easier or harder depending on the results of Section 3.
- Some questions are open-response and do not include answer choices.
- All questions allow the use of the Desmos calculator, which includes graphing functions as well as basic operations. We may also use our own calculator.

## My Essential SAT Math Strategies

- I read the question AND the answer choices before doing any work.
- I write down my work neatly on scratch paper and avoid mental math.
- I use my tools, including the Desmos calculator and the guide to formulas.
- When I'm stuck, I look for opportunities to plug in numbers and to test answer choices.
- I look for linear, quadratic, and exponential equations.
- I break word problems into individual sentences.
- I am extra careful when dealing with negative signs and when distributing.

**Solving Equations**

- Solving equations usually involves using inverse operations in order to isolate a variable.
- Pay careful attention to order of operations; simplify the equation as much as possible before using inverse operations.
- Double-check what the question asks us to find!

*1. Solve for each equation for the requested value:*

a. If $2(x + 6) - 5 = 15$, what is the value of $x$ ?

b. If $10 - 2(x + 5) = 4(2^2)$, what is the value of $5x$ ?

c. If $\frac{12}{(n-1)} = 2(7 - 5)$, what is the value of $10 - n$ ?

d. If $3(a - 7) = 2 - 3(3 - a) - 14$, what is the value of $-a$ ?

e. If $2x^2 - 11 = 4^2 + 23$, what is the value of $x^2$ ?

f. If $\frac{(x-6)}{3} = \frac{(2x+4)}{(14-23)}$, what is the value of $15x$ ?

**Creating Equations**

Creating equations is just translating a word problem into math operations.

- Look for words that have a specific mathematical meaning:
    - 'A number' is $x$ (or another variable).
    - 'Is' means equals.
    - 'Less' and 'more' mean subtract or add (for subtraction, pay careful attention to which number is being subtracted!)
- Some problems provide word problems with context; our job is to find the math statement within the word problem.

*2. Create an equation to represent each statement:*

a. Four less than a number is equal to seventeen.

b. Eighteen greater than three times a number is forty-six.

c. A number is fifteen less than sixty-two.

d. Three times a number is eight greater than thirty less than eight times the number.

e. Andre spent a total of $62, including a $7 tax, to buy 15 pool noodles that each cost n dollars.

f. Donna pays a marketing company $0.07 each time her advertisement is posted in addition to a monthly fee of $45.00. Last month her advertisement was posted p times, and her monthly bill was $132.50.

**Number of Solutions**

We are used to solving equations and finding a single answer for x. However, not all equations have one answer.

- Equations with one variable (that is NOT raised to a power) can have two solutions, one solution, or zero solutions.
- Zero solutions or infinite solutions occur when the variables cancel out when we simplify the expression.
  - If the remaining statement is true, there are infinite solutions.
  - If the remaining solution is false, there are no solutions.
- One solution occurs when we are able to isolate the variable.

*3. How many solutions does each equation have?*

a. $2x - 5 = 7 + 2(3 + x)$

b. $10 - x = 4 + x$

c. $2(5 - 4x) = -2 - 8x + 12$

d. $3x - (11 - x) = 11 + 4x$

**Inequalities**

Inequalities show that one value is greater than or less than another value and use <, >, ≤, or ≥ instead of =.

- We use inverse operations to isolate variables.
- We flip the direction of the sign when multiplying or dividing by a negative number.

*4. Simplify each expression as much as possible*:

a. $2x - 4 > 6x - 11$

b. $10(3 - x) < -2x + 9$

c. $-6(2 + x) \geq -5(3 - x)$

d. $2x + 9(5 - x) \leq 4 - (2 + x)$

**Absolute Value**

Absolute value is a measure of the distance from zero. Because it is a measure of distance, the absolute value of a number is never negative.

- If there is a variable inside the absolute value, expect to have two solutions.
    - The first step is to isolate the absolute value on one side of the equations.
    - For the first solution, remove the absolute value sign and solve.
    - For the second, remove the absolute value and multiply the other side of the equation by negative one.
- If an absolute value is equal to a negative number there will be no solutions.

*5. Find all possible values of x in each equation*:

a. $|x + 2| = 5$

b. $|6 - 2x| + 4 = 14$

c. $|3x + 4| = -5$

d. $2(4 + 5) = 3|5 - x|$

**Algebraic Rearranging**

Algebraic rearranging involves using inverse operations to solve an equation for a specific variable or value.

- These questions are easy to spot because every answer will be an equation, usually set equal to a specific variable.
- Every equation has infinite variations, so we may need to simplify answer choices to determine which one matches ours.
- Look for the phrasing 'a in terms of b'. This simply means we need to solve for a.

*6. Solve each equation for the specified value:*

a. If $y = 2x + 4$, what is $x$ in terms of $y$ ?

b. If $a + 6 = (2b - 3)/4$, what is $b$ in terms of $a$ ?

c. If $(b + c)/3 = 2n + f$, what is the value of $n$ in terms of $b$, $c$, and $f$ ?

d. If $\frac{(2d+c)}{a} = \frac{(3c-4)}{2}$, what is the value of $c$ in terms of $a$ and $d$ ?

**Linear Expressions**

The Two Essential Equations

- $y = mx + b$
  - This is called slope-intercept form.
  - 'm' represents the slope of the line. In word problems, this often represents the rate of increase or decrease.
  - 'b' represents the y-intercept of the line. This is also referred to as the original, starting, initial, or fixed value.

- $ax + by = c$
  - This is called standard form.
  - The slope can be calculated by finding the value of $-\frac{a}{b}$ or converting to slope-intercept form by solving for y.

Important Formulas to Know

- Slope: $\frac{y_1 - y_2}{x_1 - x_2}$
  - Sometimes referred to as 'rise over run' or 'change in y over change in x'.
  - Parallel lines have the same slope. Perpendicular lines have slopes that are the opposite reciprocal of one another (for example: $\frac{2}{3}$ and $-\frac{3}{2}$)

- Midpoint: $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$
  - This formula simply averages the x values and the y values of two points.

- Distance: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
  - The distance formula is a version of the Pythagorean theorem. Therefore, we can also solve for distance by drawing a right triangle.

**Creating Linear Expressions**

Some questions ask us to create a linear expression to represent a given situation. We can usually identify these questions because all of the answers will be equations of lines.

- Most questions ask us to create an equation in slope-intercept form.
- To create a linear expression, we need to know either the slope and one point on the graph or two separate points on the graph.
- Once we know the slope, or '$m$' value, we can plug in a point to find the y-intercept, or '$b$' value.
- Plugging in points helps us confirm our answer. Remember, $f(x)$ just means $y$.

*7. Create an equation in the form $y = mx + b$ for each set of given information:*

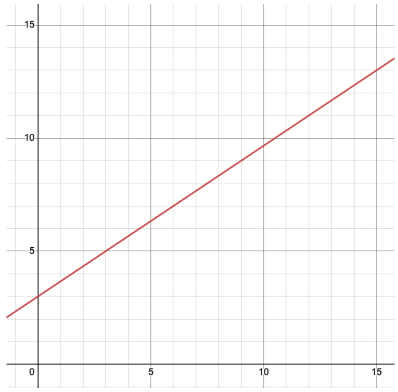
a. In the xy-plane, the graph of the linear function $f$ contains the points $(0, 7)$ and $(4, -13)$. What equation defines $f$, where $y = f(x)$?

b. In the xy-plane, the graph of the linear function $f$ contains the points $(6, -2)$ and $(-4, 8)$. What equation defines $f$, where $y = f(x)$?

c. For the linear function $f$, the table contains three values of $x$ and their corresponding values of $f(x)$. What is the equation of $f(x)$?

| | | | |
|---|---|---|---|
| $x$ | $4$ | $6$ | $8$ |
| $f(x)$ | $-13$ | $-7$ | $-1$ |

d. What is the equation of the graph shown?




e. In the linear function $f$, $f(2) = 5$ and $f(5) =- 7$. What is the equation of function $f$?

f. Linear function $f$ has a y-intercept of $-8$ and an x-intercept of 2. What is the equation of function $f$?

g. Ira rents a truck for $50 per day plus a one-time administrative fee of $35. What equation represents the total cost $c$, in dollars, to rent the truck for $d$ days?

h. A lawnmower mechanic charges a total of $150 for the first 3 hours of repair, plus an hourly fee for each additional hour. The total cost of 6 hours of repair is $270. What equation represents the total cost in dollars, $y$, for $x$ hours of repair, where $x \geq 3$?

**Solving Linear Expressions**

- Pay attention to what the question is asking for: are we solving for the slope, an intercept, or something else?
- Look for opportunities to plug in numbers. Sometimes we'll have to plug in more than one value in order to understand how an equation behaves.

*8. Use the given equations to answer each of the following questions*:

a. The equation $c = 7.5h + 10$ represents the total cost $c$, of renting a scooter for $h$ hours. If Sonya rents a scooter for 3 hours, what is her total cost?

b. The function $g(p) = 945 - 150p$ represents the number of grams of flour remaining after making $p$ pancakes. How much flour is used to make each pancake?

c. The relationship between the number of cats, $c$, and the number of dogs, $d$, that can be washed by a pet groomer on a given day is represented by the equation $8d + 12c = 120$. If the pet groomer washed 4 cats on a given day, how many dogs can they wash on the same day?

d. The function $h(t) = 252 - 12t$ models the height of a balloon $t$ seconds after it springs a leak. According to the function, how many seconds until the balloon hits the ground?

**Interpreting Linear Expressions**

Questions about interpreting linear expressions ask us about the meaning of specific coefficients, variables, integers, or combinations thereof.

- Determine whether the equation is in slope-intercept form, standard form, or something else.
- Identify the slope and/or intercepts of the line and determine what these values mean in relation to the variables in the equation.

*9. Use the given equations to answer the following questions:*

a. Tilda is taking a standardized test. The function $f(x) = -2.5x + 125$ represents the amount of time, in minutes, that Tilda has remaining after completing $x$ questions. What is the meaning of the y-intercept of the graph of $y = f(x)$ in the xy-plane in this context?

b. Last month, a social media manager earned $1,860 from writing content for $x$ hours and editing videos for $y$ hours. The equation $35x + 45y = 1860$ represents this situation. What is the meaning of $45$ in this context?

c. A factory produces square and rectangular tiles. The equation $2x + 3y = 120$ represents the total number of square tiles, $x$, and rectangular tiles, $y$, that the factory can produce in a single day. In this context, what is the meaning of $(x, y) = (15, 30)$?

## Systems of Linear Equations

When we have multiple equations that use the same variables, we call this a system of equations. Linear means that the equations represent lines.

- If we have two linear equations, there are three possibilities:
  - There is one point of intersection. This means that the two lines have different slopes. Their y-intercept is irrelevant.
  - There are zero points of intersection. This means that the two lines have the same slope (which means they are parallel), but different y-intercepts.
  - There are infinite points of intersection. This means that the two lines have the same slope AND the same y-intercept. They are the same line!
- Be sure to fully-reduce fractional slopes; $\frac{3}{6}$ and $\frac{7}{14}$ are equal slopes because they both reduce to $\frac{1}{2}$.

*10. How many solutions are there for each of these systems of equations?*
*(Our options are zero, one, and infinity)*

a.

$y = (\frac{2}{3})x + 6$

$y = (\frac{8}{12})x - 5$

b.

$2x + 4y = 12$

$1x + 2y = 6$

c.

$2x + 5y = 3$

$y = 4x - 1$

d.

$y = (\frac{1}{2})x - 3$

$y = (-2)x + 2$

e.

$8x - 5y = 11$

$24x - 15y = -17$

f.

$y = x - 8$

$x - y = -8$

g.

$y = (\frac{5}{6})x - 7$

$y = (\frac{15}{18})x - 7$

h.

$5x + 3y = 12$

$3y - 5x = 12$

i.

$7x + 6y = 19$

$y = -(\frac{7}{6})x + 19$

**Solving Systems of Equations**

- There are three primary ways to solve systems of equations: graphing, substitution, and elimination.
    - With graphing, we enter the equations into the Desmos calculator and find the point(s) of intersection.
    - With substitution, we isolate a variable in one equation, then we substitute our answer into the other equation.
    - With elimination, we manipulate one equation so that the coefficients of one variable match the coefficient in the other equation, then we add or subtract the two equations.
- Graphing is a great tool for confirming our answer.

*11. Which method would we use to solve each of these systems of equations?*

a.

$2x + 3y = 18$

$x = 8 - y$

b.

$4x - 3y = 11$

$2x + 4y = 22$

c.

$5x - 7y = -23$

$x + y = 5$

When solving for a relationship between numbers (such as $x - y$ or $3y + x$), start by looking for a shortcut by adding and subtracting the two equations.

*12. Find a shortcut to solve the equations*:

a. If $2x + 5y = 11$, and $x + 4y = 19$, what is the value of $x + y$?

b. If $x - y = 19$ and $3x + 5y = 61$, what is the value of $2x + 2y$?

**Creating Systems of Equations**

- Word problems that contain two variables or unknown values are usually solved by creating a system of equations.
- On the SAT, systems of equations word problems usually ask about linear expressions written in standard form.
- Look for common units when creating equations for these problems, such as money, time, or quantity.
- Use variables that match the names of what they represent. Be careful when using letters that can look like numbers.

*13. Create and solve equations that represent each of these situations*:

a. Tina goes to a bookstore where softcover books cost $4 and hardcover books cost $9. Tina buys 7 books and spends $38. How many softcover books did Tina buy?

b. Jayson Tatum made a total of 15 shots in his last game. All of his shots were worth either 2 points or 3 points. If he scored a total of 34 points, how many three pointers did he make?

c. Gulnaz makes $14/hour and Nick makes $12/hour. Last Saturday, Gulnaz and Nick made a total of $132. Gulnaz worked 2 hours more than Nick. How much money did Nick make on Saturday?

**Creating Systems of Inequalities**

- Systems of inequalities are just like other systems of equations problems, but they use inequality signs instead of equal signs.
- The following words indicate an inequality:
  - 'Greater than' or 'less than'
  - 'At least' or 'at most'
  - 'No less than' or 'no more than'
  - 'Up to'
- Like systems of equations, systems of inequalities can be graphed in the Desmos calculator.

*14. Which sign would correctly complete the equations below?*

a. Martin makes \$20/hour for painting houses and \$15/hour for mowing lawns. Martin will make at least \$400 dollars this week, and he will work no more than 25 hours.

$20h + 15m$ __ $400$
$h + m$ __ $25$

b. Serena is buying food for the tigers and zebras at a zoo. Each bag of tiger food weighs 50 pounds, and each bag of zebra food weighs 20 pounds. Serena will buy no less than 15 bags of food, but she can only transport up to 500 pounds of food in her car.

$t + z$ __ $15$
$50t + 20z$ __ $500$

*15. What equations would represent the situation below?*

a. On a particular test, each multiple-choice question is worth 5 points while each fill-in-the-blank question is worth 7 points. There are no more than 20 questions on the test, and the test is worth a total of at least 110 points.

**Solving Systems of Inequalities**

We solve systems of inequalities by either plugging in coordinates or by graphing the quotations and finding points in the xy plane.

- If we plug in coordinates and the resulting expression is true, then the coordinates are part of the solution set.
- Graphing inequalities can take more time because of the additional symbols required, but this method is a great way to confirm our answer, especially when the numbers are difficult.

*16. Use plugging-in or graphing to answer the following questions*:

a. Given the system of inequalities below, circle the points that are part of the solution set to this inequality:

$y < 2x - 5$

$y > -\frac{1}{2}x + 1$

a. $(0, 0)$ b. $(2, 2)$ c. $(-6, -2)$

d. $(8, 0)$ e. $(20, -6)$ f. $(4, -1)$

b. Given the system of inequalities below, circle the points that are part of the solution set to this inequality:

$y \geq 3x + 4$

$y < -x - 3$

g. $(0, 0)$ h. $(-3, 2)$ i. $(0, 5)$

j. $(-5, 0)$ k. $(-2, -1)$ l. $(0, -1)$

**Identifying Quadratic Expressions**

- Quadratic expressions include a variable to the second power. No other variables are raised to a higher power.

*17. Circle the quadratic equations below.*

| | | |
|---|---|---|
| a. $y = 5x - 2$ | b. $y = x^2 + 3x - 5$ | c. $y = 2x^2 - 4$ |
| d. $a - 3b = 14$ | e. $2x - 4y = 11$ | f. $x(x + 4) - 5 = y$ |
| g. $2n^2 + 4n - 7 = k$ | h. $(x + 2)(x - 1) = 3$ | i. $2(x - 1) + 6 = y$ |
| j. $-(x + 4)^2 - 1 = y$ | k. $(x + y)(x - y) = 11$ | l. $(2 - x)(3 + x) = y$ |

**Properties of Parabolas**

- The graph of a quadratic expression is a parabola.
- Parabolas are U-shaped. They open upwards if the coefficient of $x^2$ is positive, and they open downward if the coefficient of $x^2$ is negative.

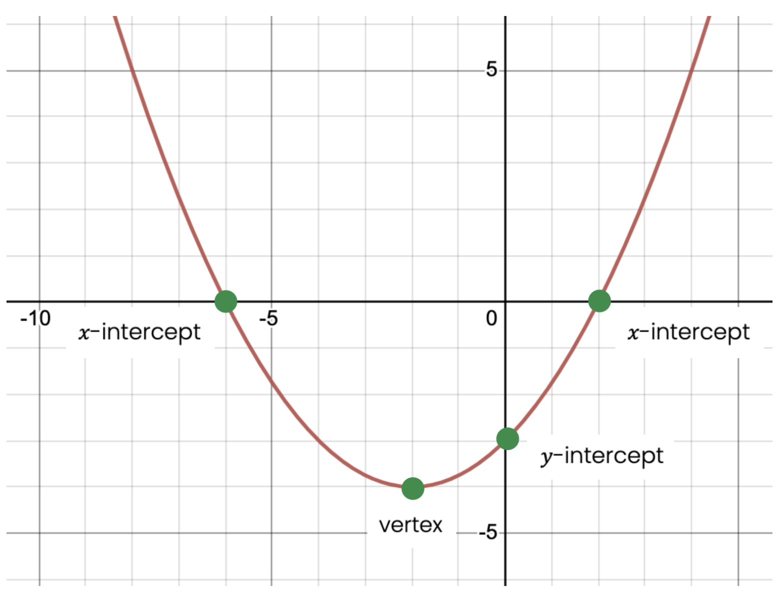
*18. Which of the parabolas on the previous page open upwards, and which ones open downwards?*

a. Upwards: ___________________________

b. Downwards: _________________________

- Parabolas have either 0, 1, or 2 x-intercepts. The x-intercepts are also called the zeros, roots, or solutions. We can solve for the x-intercepts by setting the factors equal to 0 and solving for x.
- All parabolas have a vertex. This point is also referred to as the maximum or the minimum. We can find the vertex by using the axis of symmetry or by averaging the x-intercepts.

The anatomy of a parabola:



**FOILing and Factoring Quadratic Expressions**

- **F**irst **O**uter **I**nner **L**ast, or **FOIL**ing is when we multiply two binomials together to create a quadratic equation in standard form.

*19. FOIL these quadratic equations*:

a. $(x + 6)(x - 4) = y$ b. $(3x + 2)(x - 1) = y$ c. $(x - 3)(x - 7) = y$

- Factoring is when we take a quadratic in standard form and expand it to create two binomials with the format $(x - p)(x - q)$ where $p$ and $q$ are the $x$ -intercepts.
- For a quadratic with the equation $y = ax^2 + bx + c$ where $a = 1$, it will be true that $p + q = b$ and $pq = c$.

*20. Factor these quadratic equations*:

a. $x^2 + 5x + 6 = y$ b. $x^2 - 3x - 4 = y$ c. $x^2 + 12x - 28 = y$

- Once an equation is in factored form, we can find the $x$-intercepts by setting each of the factors equal to zero and solving for $x$.

*21. Factor these quadratic equations and solve for the $x$-intercepts*:

a. $x^2 + 11x + 30 = y$ b. $x^2 - 10x + 25 = y$ c. $x^2 + 7x - 18 = y$

- When $a \neq 1$, we factor by finding the product $ac$ and then identifying factors that add to $b$. From here, we rewrite the equation and factor by grouping.

*22. Factor these quadratic equations and solve for the $x$-intercepts:*

a. $2x^2 + 11x + 12 = y$ b. $3x^2 - 16x + 5 = y$ c. $12x^2 + 24x + 12 = y$

- When an equation appears to be a quadratic but is not in the form $ax^2 + bx + c$, we must simplify the equation into this form before we attempt to factor.

*23. Simplify these quadratic equations equations, factor, and solve for the $x$-intercepts:*

a. $x^2 + 3x = y + 4x$ b. $7x - 17 = y - x^2 + 1$ c. $3x^2 + x = y + 2x + 4$

**Differences of Squares**

- Differences of squares occur when two binomials with opposite signs are multiplied together.

*24. FOIL these quadratic equations*:

a. $(x + 3)(x - 3) = y$  b. $(x + 5)(x - 5) = y$  c. $(x - 10)(x + 10) = y$

- We can spot differences of squares when two perfect squares are joined by subtraction, and we can factor them by using the square roots of each term.

*25. Factor these quadratic equations*:

a. $x^2 - 16 = y$  b. $x^2 - 49 = y$  c. $x^2 - 36 = y$

**Matching Corresponding Terms**

Matching corresponding terms is when we solve for a coefficient or integer by comparing two equal expressions.

- First, put both terms in $ax^2 + bx + c$ form in order to easily identify the value of $a$, $b$, and $c$.
- It is important that $y$ is isolated when identifying $a$, $b$, and $c$.

*26. Identify the $a$, $b$, and $c$ terms in each of the following quadratic expressions*:

a. $3x^2 + 16x - 9 = y$

b. $x^2 - 11x + 49 = y - 4$

c. $5x^2 + (2 - u)x + 36 = y$

d. $-x^2 + kx = y$

e. $x^2 - 49 = y + 6x$

f. $2x^2 + 4x - 7 + t = y$

- When two equal equations are in the same form, we can set their $a$, $b$, and $c$ values equal to one another.

*27. Answer the following questions*:

a. If $(x + 7)(x + t) = x^2 + kx + 21$, what is the value of k?

b. Given that $x^2 + 11x + v = (x + h)(x + 12)$, what is the value of $h + v$?

c. The expression $(2x + 5)(x + w)$ is equivalent to the expression $2x^2 + 13x + n$. What is the value of $n$?

**Finding the Vertex**

- To find the vertex of a parabola, first find the x value of the vertex, which is also the axis of symmetry, by either:

  • Putting the equation in standard form, then finding $-\frac{b}{2a}$.

  • Putting the equation in factored form, then averaging the $x$-intercepts.

- Next, plug in the $x$ value of the vertex to find the $y$ value of the vertex.

*28. Find the vertex of these quadratic expressions*:

a. $x^2 - 4x + 21 = y$

b. $(x - 5)(x + 3) = y$

c. $x^2 - 36 = y$

## Identifying Forms of Quadratic Expressions

There are three forms of quadratic expressions, each of which have different properties:

- Standard Form: $ax^2 + bx + c = y$
    - $a$ tells us the direction.
    - $(0, c)$ is the y-intercept.
    - $-\frac{b}{2a}$ is the axis of symmetry.
- Factored Form: $a(x - p)(x - q) = y$
    - $a$ tells us the direction.
    - $(x - p)$ and $(x - q)$ are the factors.
    - p and q are the roots, solutions, zeros, or x-intercepts
- Vertex Form: $a(x^2 - h) + k = y$
    - $a$ tells us the direction.
    - $(h, k)$ is the vertex.

*29. Which form is each of the following quadratics in?*

a. $x^2 - 4x + 21 = y$

b. $(x - 5)^2 + 1 = y$

c. $(x - 5)(x + 3) = y$

d. $-7(x + 6)^2 - 12 = y$

e. $x^2 - 9 = y$

f. $x(x + 4) - 5 = y$

**Solving Systems of Equations with Quadratic Expressions**

Some systems of equations problems will include a quadratic expression. In these situations, we will still use graphing, substitution, or elimination to combine the equations and solve for the variable.

- Pay attention to which variable we need to solve for. Sometimes we will need to find one value and then plug in to find the other.

*30. Answer the following questions using substitution, elimination, and factoring*:

a. Given the equations $y = x^2 + 2x$ and $y = x + 6$, what are the possible values of $x$?

b. If $y = 3x + 5$ and $y = -2x^2 - x + 11$, what are the possible values of $y$?

c. Given that $y = 3x + 15$ and $y = x^2 + 4x - 15$, if $x > 0$, what is the value of $y$?

**The Quadratic Formula**

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The quadratic formula can be used to solve several types of questions. While the equation as a whole allows us to solve for the x intercepts, it also helps us to determine the number of solutions, the axis of symmetry, and the vertex.

- In order to use the quadratic formula, we first must put the equation in standard form.
- The most common use of the quadratic formula is to solve for the x intercepts when an equation cannot be factored.

*31. Solve each equation for x using the quadratic formula:*

a. $y = x^2 + 6x - 3$

b. $y = 2(x - 5)^2 - 4$

c. $y - 3 = (x - 2)(x + 4)$

The quadratic formula is actually made up to two components: the axis of symmetry ($-\frac{b}{2a}$) and the discriminant ($b^2 - 4ac$)

- The axis of symmetry allows us to find the $x$ value of the vertex; we can plug this value into the equation to find the $y$ value of the vertex.
- The discriminant tells us the number of solutions:
  - If the discriminant is positive, there are two real solutions.
  - If the discriminant is equal to zero, there is one real solution.
  - If the discriminant is negative, there are zero real solutions.

*32. Use the axis of symmetry or the discriminant to answer the following questions:*

a. What is the x value of the vertex of the equation $y = 3x^2 + 12x - 20$ ?

b. How many x-intercepts does the function $f(x) = x^2 - 12x + 38$ have?

c. How many real solutions are there to the equation $y = 6x^2 - 24x + 24$ ?

d. What is the maximum value of $f(x)$ for the function $f(x) = -2x^2 + 8x + 13$ ?

e. How many roots are there for the function $f(x) = 2x^2 + 3x + 10$ ?

f. If the equation $y = 3x^2 + bx + 3$ has one real solution, what is a possible value of $b$?

**The Sum of Solutions**

The sum of solutions to a quadratic equation can be written as -b/a. In order to use this formula, the equation must be in standard form.

*33. Find the sum of the solutions for the following equations:*

a. $y = 2x^2 + 6x - 21$

b. $y = 7x^2 - 8x + 19$

c. $y - 4x = x^2 + 7x - 11$

d. $x^2 + y = 3x^2 - 10x + 1$

## Functions and f(x) Notation

$f(x)$ notation allows us to represent inputs and outputs of equations more efficiently.

In $f(x)$ notation:

- $x$ is the input.
- $f(x)$ is the output or $y$ value.
- The input and output for an $f(x)$ function make up a coordinate pair.
- $f(x)$ is often set equal to an equation; once we know this equation, everytime we see $f(x)$ we can assume we're using the same equation.

*34. Solve the following questions based on these functions*:

$$f(x) = 2x - 7$$

$$g(x) = x^2 + 6$$

a. $f(3) =$

b. $g(9) =$

c. $f(22) =$

d. $g(n) = 31$, find $n$

e. $f(f(4)) =$

f. $g(f(8)) =$

g. $f(k) = 11$, find $k$

h. $f(g(m)) = 13$, find $m$

i. $f(2v) = 17$, find $v$

j. $g(p) = 55$, find $p$

## f(x) Tables

Sometimes the SAT will not give us an equation but will instead present us with a table of values. The tables usually have two columns:

- The $x$ column shows the input.
- The $f(x)$ column shows the output or y values.
- Each row in the column is a coordinate pair.

Some tables have three or more columns. These tables represent multiple functions at the same time. The inputs will be the same for both functions, while the outputs will usually be different.

*35. Solve the following questions based on this f(x) table:*

| x | f(x) | g(x) |
|---|---|---|
| $-2$ | $-6$ | $10$ |
| $-1$ | $-2$ | $-2$ |
| $0$ | $5$ | $0$ |
| $1$ | $3$ | $6$ |
| $2$ | $-4$ | $6$ |

a. $f(1) =$

b. $g(-2) =$

c. $f(0) =$

d. $g(m) = 0$, find $m$

e. $g(f(-1)) =$

f. $f(n) = -2$, find $n$

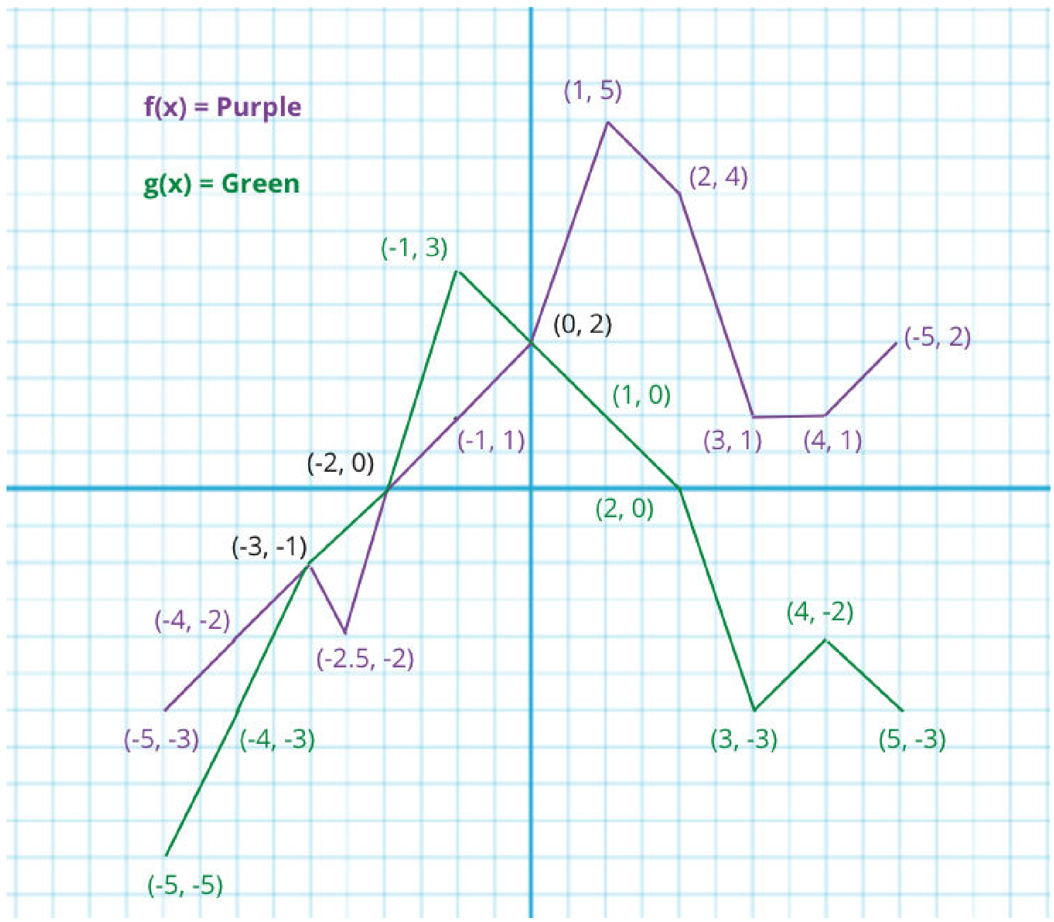
g. $f(k - 1) = -4$, find $k$

h. $h(x) = f(2x) + g(x - 2)$, find $h(1)$

## f(x) Graphs

Functions can also be represented visually. If we know an $(x, y)$ coordinate, we can then create a true $f(x) \ = \ y$ statement.

*36. Solve the following questions based on this $f(x)$ graph:*




a. $f(4) =$

b. $g(-\ 2) =$

c. $f(p) = 5, \ find\ p$

d. $f(g(3)) =$

e. $g(n) = 0$, find $n$

f. $g(n) = f(n)$ for what values of $n$

g. $f(3) \ + \ g(3) =$

h. $g(x) \geq f(x)$ for what values of $x$

**Exponent Operations**

- There are just six exponent rules we need to know for the SAT.
  - $x^a \bullet x^b = x^{(a+b)}$
  - $\frac{x^a}{x^b} = x^{(a-b)}$
  - $(x^a)^b = x^{(ab)}$
  - $x^{-a} = \frac{1}{x^a}$
  - $x^{\frac{a}{b}} = \sqrt[b]{x^a}$
  - $x^0 = 1$
- Manage complex problems by doing just one step at a time.

*37. Simplify the following expressions*:

a. $(x^{11})^2 \bullet x^5$

b. $4x^{-\frac{1}{2}}$

c. $\frac{(2w^{\frac{4}{5}})^5}{w}$

d. $\frac{x^4y^6}{xy^2}$

e. $3(2x^3)^2 + 4x^4$

f. $\frac{a^{-2}b^0c^{\frac{2}{3}}}{a^6b^{\frac{-1}{2}}c^{-1}}$

**Radical Operations**

- Simplify radicals by finding perfect squares or cubes that are factors of the number under the radical.

*38. Simplify the following expressions*:

a. $\sqrt{128}$

b. $\sqrt[3]{27}$

c. $\sqrt[3]{32}$

d. $\sqrt{27}$

e. $4\sqrt{75}$

f. $\sqrt{800}$

g. $\sqrt{x^4}$

h. $6x\sqrt{25x^8}$

i. $\sqrt{98x^3y^5}$

- We can undo radicals using exponent operations such as squaring or cubing both sides of the equation.
- When squaring radicals, we have to look out for extraneous roots. We can find these by plugging our answers into the original equation.

*39. Solve the following expressions and identify any extraneous roots*:

a. $x + 2 = \sqrt{x + 14}$

b. $\sqrt{x - 1} + 4 = x - 3$

c. $\sqrt{-2x + 30} = x - 3$

## Exponential Growth Graphs

Exponential growth graphs show the relationship between $x$ and $y$ when $x$ is an exponent.

- $y = ab^x$ is the basic equation for exponential growth.
  - $b$ is the base.
  - In this form, the y-intercept is $a$.
- When $a$ is negative, the equation flips over the $x$ axis.
- When $x$ is negative, the equation flips over the $y$ axis.

*40. Match each graph to the equation it represents:*

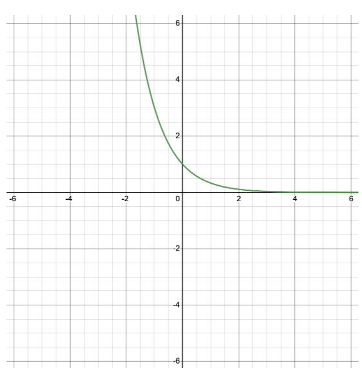
a. $y = 2^x$ is graph ___.

b. $y = -3^x$ is graph ___.

c. $y = 3^{-x}$ is graph ___.

d. $y = -2^{-x}$ is graph ___.

1.



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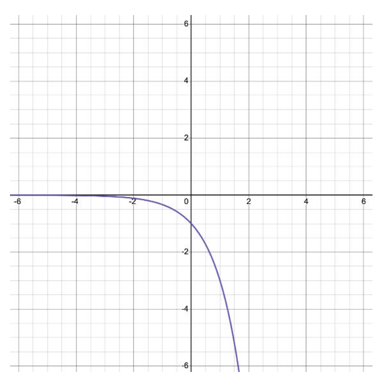

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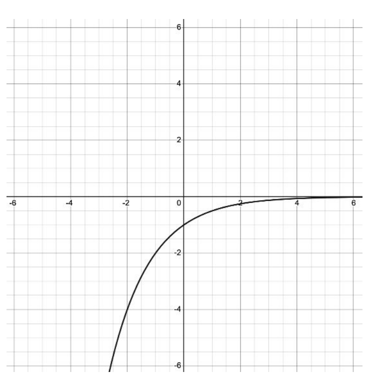

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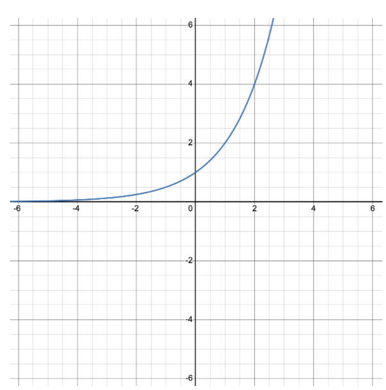


- $y = a(b)^{(x-h)} + k$ is the full equation, but it includes a multiplier as well as a horizontal and vertical transformation.
  - $b$ is still the base.
  - $a$ is the multiplier.
  - $h$ is the horizontal shift.
  - $k$ is the vertical shift.

*41. Match each graph to the equation it represents:*

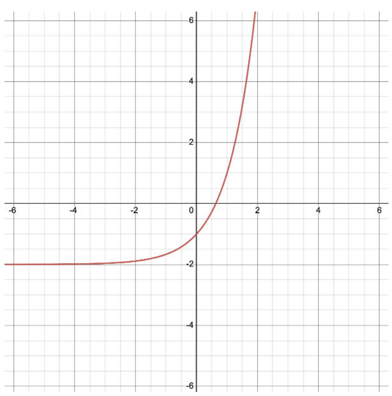
a. $y = 2(2^x)$ is graph ___.

b. $y = 3^x - 2$ is graph ___.

c. $y = 5^{(x+1)}$ is graph ___.

d. $y = \frac{1}{2} \bullet 2^{(x)} - 1$ is graph ___.

1.



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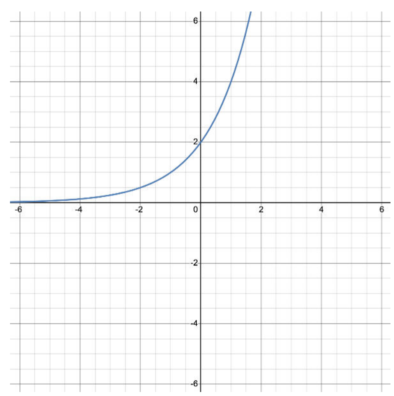

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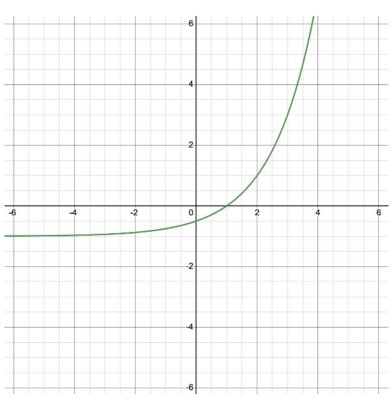

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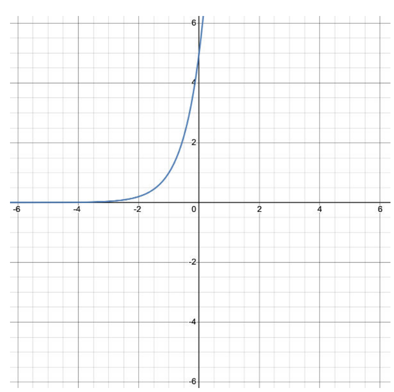


**Percents**

- The word ‘percent’ literally means divided by one hundred.
- Percent questions are best solved using decimals but can also be solved using fractions.
- In percent word problems, vocabulary is key:
    - ‘Of’ means multiplied by.
    - ‘Is’ means equals.
- The answers to percent questions should be logical. Take a moment to do a ‘gut check’ about whether an answer makes sense.

*42. Write an equation that represents each of the following statements, then solve:*

a. 38 is 19% of what number?

b. What is 14% of 150?

c. 44 is what percent of 220?

d. 30% of what number is 75?

e. 118% of 50 is what number?

**Percent Change**

- To calculate percent change, we must add or subtract the percent (as a decimal) from 1.
- When there is more than one percent change, each step must be completed separately.

*43. Write an equation that represents each of the following statements, then solve:*

a. 25 is increased by 20%.

b. 40 is increased by 5%.

c. 220 is decreased by 30%.

d. 76 is increased by 150%

e. 50 is increased by 10%, then decreased by 20%.

f. 350 is decreased by 60%, then increased by 40%.

g. 140 is increased by 15%, then increased by 5%.

h. 200 is decreased by 30%, then increased by 110%.

- Some questions ask us to find the starting value, which means using division instead of multiplication.

*44. Write an equation that represents each of the following questions statements, then solve:*

a. What number, when increased by 40%, is equal to 280?

b. A number is increased by 25%, then decreased by 50%, resulting in 75. What is the starting number?

c. The number 250 is first increased by 12%, then it is decreased by p percent. The resulting number is 224. What is the value of p?

**Exponential Growth and Decay**

Exponential growth equations follow a consistent format, including a variable in the exponent.

- $y = a \bullet b^{x}$
  - $a$ is the starting or initial value; this number is the $y$-intercept.
  - $b$ indicates the rate of increase or decrease as a decimal. This value can be thought of as $1 \pm r$ where $r$ is the rate of increase or decrease as a decimal. For example, a 5% increase would have an $r$ value of .05, and $b$ would be 1.05.
  - $x$ usually represents time. When $x$ is part of a fraction, find what value makes the exponent equal to one. This is the amount of time it takes for the percent change to occur once.

*45. Write an equation that represents each of the following statements:*

a. Sondra puts $1,500 in a savings account that earns 4% interest each year.

b. The population of Boston, MA is about 650,000, and the population increases by a factor of 1.1 every 8 years.

c. Joel purchases an RV for $40,000. The RV loses 5% of its value every 2 years. Represent the value of the RV $q$ <u>quarters</u> after Joel purchases it.

d. Scientists have acquired 140 grams of a newly discovered isotope that has a half-life of 245 years.

**Mean, Median, Mode, & Range**

When given a data set, either as a list of numbers or a graph or table, we will often have to find one of the four most common measures of the set: the mean, the median, the mode, or the range.

- The mean of the data set is often called the average. It can be found by adding up all the elements (or numbers) in the set and then dividing by the total number of elements.
- The median of the data set is the middle element when the numbers are listed from least to greatest. When a data set has an even number of elements, the median is the average of the two middle values.
- The mode of the data set is the most common element. Some data sets have more than one mode, and others do not have a mode at all.
- The range of the data set is the difference between the largest and smallest elements.

*46. Find the mean, median, mode, and range of each data set:*

| a. | b. | c. |
| --- | --- | --- |
| 2, 3, 5, 5, 7, 18, 20 | -2, 11, 19, 21, 20 | -90, -90, -50, -45, -45, -30, -20, 0 |
| Mean: ____ | Mean: ____ | Mean: ____ |
| Median: ____ | Median: ____ | Median: ____ |
| Mode: ____ | Mode: ____ | Mode: ____ |
| Range: ____ | Range: ____ | Range: ____ |

- Some data sets will be presented as histograms or tables. In histograms, which look a lot like bar graphs, the y-axis represents the number of times an element shows up in the data set. In tables, this is referred to as frequency.

*47. Use the tables and graphs to find the mean, median, mode, and range*:

| Temperature | Frequency |
|---|---|
| 68° | 4 |
| 69° | 6 |
| 70° | 3 |
| 71° | 7 |
| 72° | 8 |

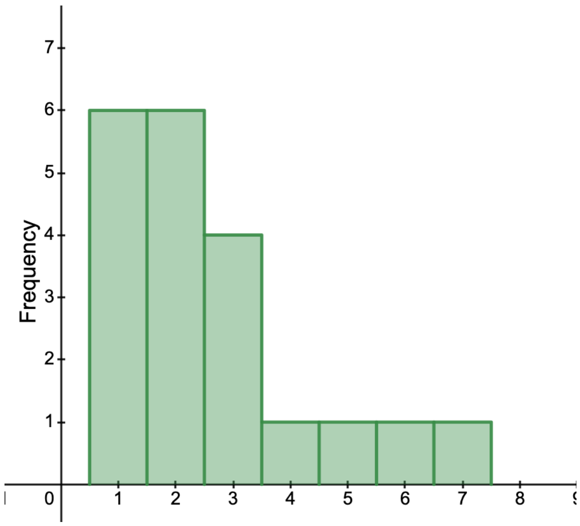
a. The table above shows the frequency of high temperatures over a span of 28 days.

Mean: ____
Median: ____
Mode: ____
Range: ____

| Number of Pets | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Number of Students | 16 | 34 | 23 | 9 | 12 | 6 |

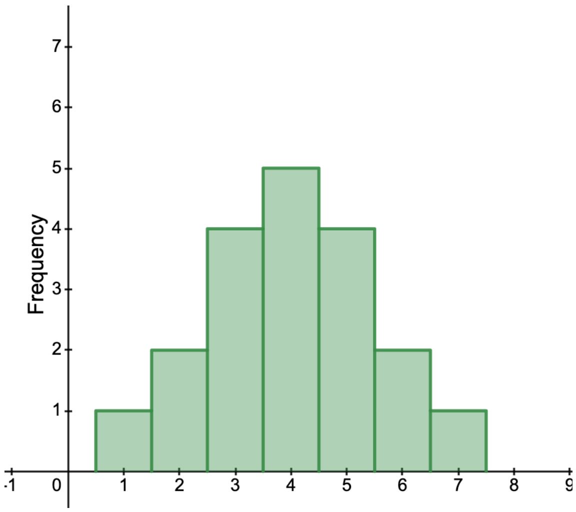
b. The table above shows the results of a survey of 100 students who were each asked how many pets they have.

Mean: ____
Median: ____
Mode: ____
Range: ____



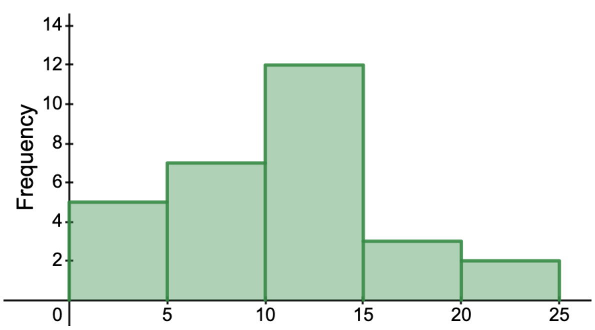

c. The histogram above shows the results of a survey in which students in a 5th grade class were asked how many siblings they have.

Mean: ____
Median: ____
Mode: ____
Range: ____




d. The histogram above shows the results of a survey in which 19 first-year college students were asked how many AP classes they took in high school.

Mean: ____
Median: ____
Mode: ____
Range: ____




e. The histogram summarizes the distribution of a data set composed of 29 integers. The first bar represents the number of integers that are at least 0 but less than 5. The second bar represents the number of integers that are at least 5 but less than 10, and so on. What is one possible value of the median of the data set?

**Standard Deviation**

Standard deviation is a measure of how closely grouped or spread out the numbers in a data set are. The standard deviation of a data set is calculated by finding the average of the distance from each data point to the mean.

- A low standard deviation indicates that the numbers are closely grouped. The values will likely have small differences between them, and the set will not have many outliers or extreme values.
- A high standard deviation indicates that the numbers are spread out. There will be greater differences between the data points, and they may be 'polar', or grouped around two distinct values.
- The SAT does not require us to calculate standard deviation by hand. Most questions can be answered by recognizing the concept of spread (as with the examples below) or by using the Desmos to calculate the precise value.

*48. For each pair of data sets, indicate which has a greater standard deviation*:

a.
Set A: 3, 4, 4, 5, 7
Set B: 2, 4, 8, 22, 23

b.
Set A: 1, 3, 5, 7, 9
Set B: 1, 2, 3, 4, 5

c.
Set A: 1, 1, 1, 1, 1, 2, 2, 3, 4, 4, 4, 5, 5, 5, 5
Set B: 2, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6

d.
Set A: 4, 6, 8, 10, 12
Set B: 5, 7, 9, 11, 13

**Proportions**

Proportions express the relationship between two values, and they are an important tool in helping us solve questions about ratios or rates. We will typically answer these questions by creating two equal fractions and cross-multiplying to solve for the unknown value.

- Look for questions that include specific units, such as gallons, meters, hours, or seconds. The units will typically relate to size, distance, or time.
- Proportions will be used to solve questions about unit conversions. Corresponding units should NOT be at a diagonal when we set up the proportion.

*49. Use proportions to answer the following questions*:

a. It takes Rodger 6 hours to spread mulch over four hectares. How many hours will it take Rodger to spread mulch over 13 hectares?

b. Marta is able to address envelopes at a rate of 270 envelopes per hour. How fast can Marta address envelopes in envelopes <u>per minute</u>?

c. In a random sample of students from a particular high school, 16 out of 50 students reported having done their homework on the bus. If there are 243 students at the school, approximately how many students can be expected to have done their homework on the bus? Round the answer to the nearest whole number.

d. Nichol can paint three rooms in 4.5 hours. David can paint 4 rooms in 7 hours. How many fewer <u>minutes</u> does it take for Nichol to paint a room? Round the answer to the nearest whole number.

**Probability**

- $\frac{number\ of\ successes}{number\ of\ possible\ outcomes}$ is the formula for calculating simple probabilities.
- Probabilities can be represented as fractions or decimals. The sum of probabilities must be equal to 1.
- Probability questions with tables often include more information than we need.

*50. Answer the following questions*:

a. A gumball machine contains 45 red gumballs, 26 blue gumballs, 38 yellow gumballs, 12 purple gumballs, and 51 green gumballs. If a gumball is selected at random, what is the probability that it is either red or blue?

b. There are 540 attendees at a minor league baseball game. One attendee is selected at random. The probability that an attendee is 0-15 years old is 0.35, and the probability that they are 16-35 years old is 0.20. How many of the attendees are 36 years or older?

c. The table below shows the number of right-handed pitchers and the number of left-handed pitchers in two college baseball divisions. If a pitcher from these two divisions is chosen at random, what is the probability that they are a left-handed pitcher in the New England Division?

| Division | Right-Handed Pitchers | Left-Handed Pitchers |
| --- | --- | --- |
| Mid-Atlantic Division | 215 | 56 |
| New England Division | 168 | 40 |

d. The table below summarizes the voting behavior of participants in a local election. If a participant between the ages of 18 and 45 is chosen at random, what is the probability that they voted for candidate C?

| **Age Group** | **Candidate A** | **Candidate B** | **Candidate C** |
| --- | --- | --- | --- |
| 18-30 | 206 | 45 | 107 |
| 31-45 | 287 | 94 | 123 |
| 46-60 | 165 | 196 | 204 |
| 61+ | 117 | 206 | 305 |
| **Total** | 775 | 541 | 739 |

**Margin of Error**

Margin of error provides us with a range of plausible values.

- Margin of error questions provide an estimated value and a margin of error.
- Add and subtract the margin of error from the estimated value to find the range of plausible values.
- The range created by the margin of error does NOT always include the actual value. Plausible values are determined by measuring a portion of a population. The actual value would be determined by measuring the entire population.

*51. Answer the following questions*:

a. It is estimated that candidate A will receive 53.7% of the vote in an upcoming election. The margin of error is 3.5%. What is the range of plausible values for the percent of the vote received by candidate A?

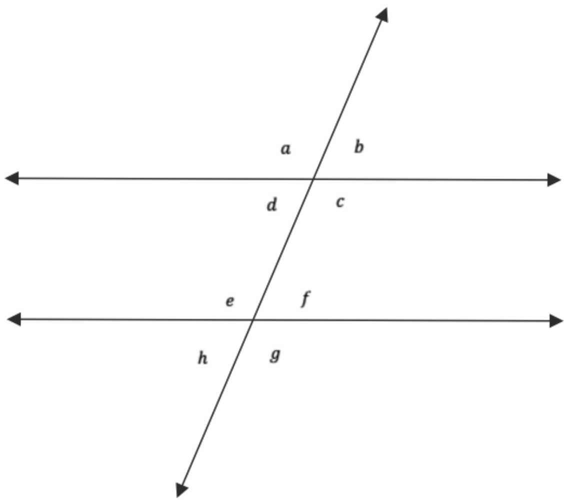
b. The mean mass of a random sample of carrots grown at a farm in Northampton is 124 grams with an associated margin of error of 27 grams. Is 172 grams a plausible value for the true mean mass of a carrot grown at this farm? Why or why not?

**Angles**

Angles questions typically ask about the angles created when two lines intersect or when two parallel lines are intersected by a transversal.

- When a transversal intersects two parallel lines, every pair of angles are either equal or supplementary (they add up to 180º).
- Possible angle relationships include:
    - Vertical angles (equal)
    - Corresponding angles (equal)
    - Opposite interior angles (equal)
    - Opposite exterior angles (equal)
    - A linear pair (complementary)
    - Same side exterior angles (supplementary)
    - Same side interior angles (supplementary)
- For questions about parallel lines, we can usually determine which angles are equal simply by looking at the diagram.

*52. Based on the diagram below, identify a pair of angles that are:*




a. Opposite interior angles: ______

b. Vertical angles: ______

c. Same side interior angles: ______

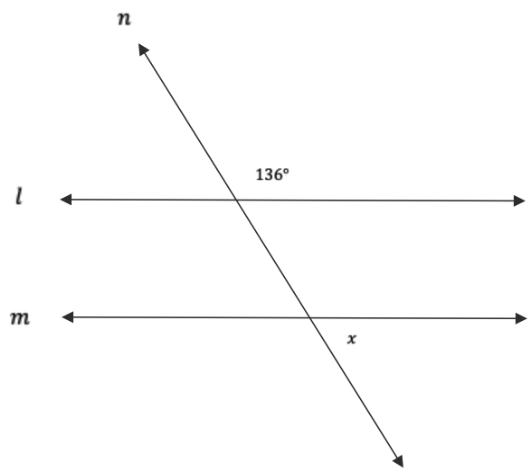
d. Corresponding angles: ______

e. Same side exterior angles: ______

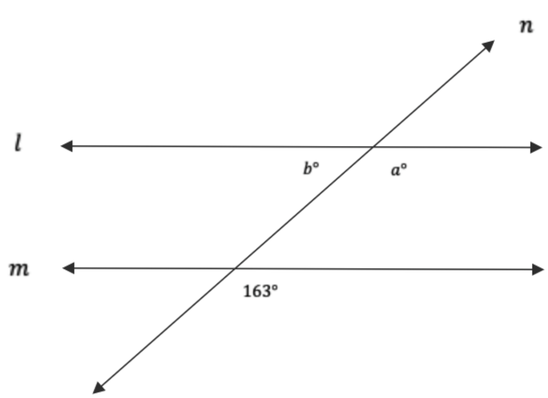
f. A linear pair: _____

g. Opposite exterior angles: ______

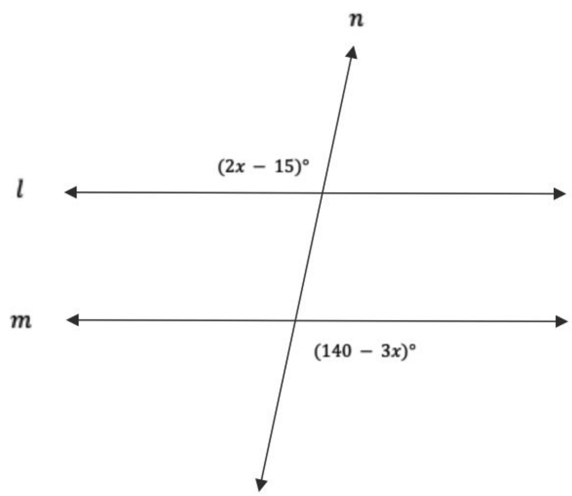
*53. Answer the following questions using the diagrams provided:*



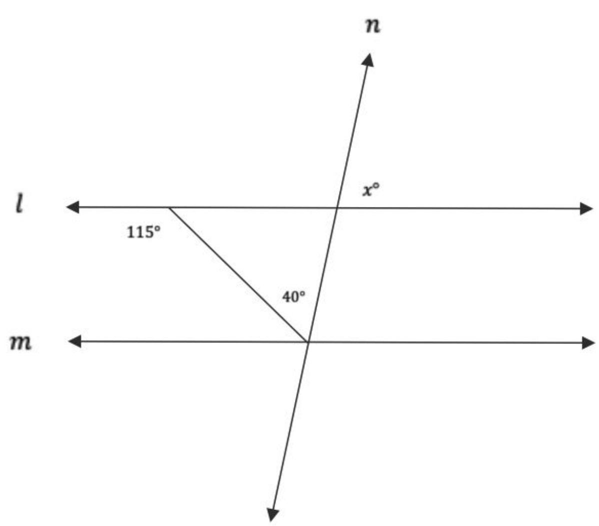

a. Given that line $l$ is parallel to line $m$, what is the measure of angle $x$?




b. Given that line $l$ is parallel to line $m$, what is the value of $a - b$?




c. If line $l$ is parallel to line $m$, what is the value of $x$?



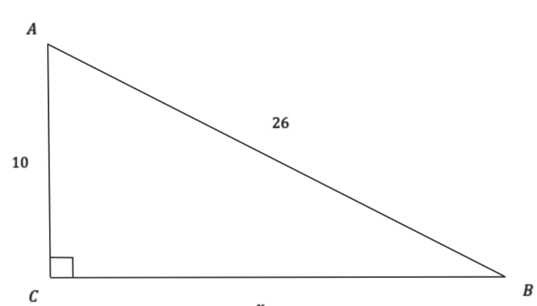

d. Given that line $l$ is parallel to line $m$, what is the value of $x$?

**Triangles**

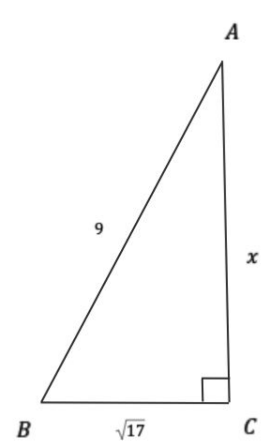
- There are four types of triangles we need to know for the SAT:
    - Scalene triangles: all angles are different and the sides are different lengths
    - Isosceles triangles: two angles are equal and their two opposite sides are equal
    - Equilateral triangles: all angles are 60° and all sides are equal
    - Right triangles: one angle is 90° and the sides complete the Pythagorean theorem; right triangles can be scalene or isosceles.
- The most important triangle formula is the Pythagorean theorem: $a^2 + b^2 = c^2$
    - The Pythagorean Theorem only applies to right triangles.
    - The SAT frequently uses Pythagorean triplets, which are trios of numbers that complete the Pythagorean theorem. The most common Pythagorean triplets are:
        - 3-4-5 (which can be scaled up to 6-8-10, 9-12-15, and so on)
        - 5-12-13
        - 7-24-25
        - 8-15-17

*54. Solve for $x$ in each of these right triangles:*

a.



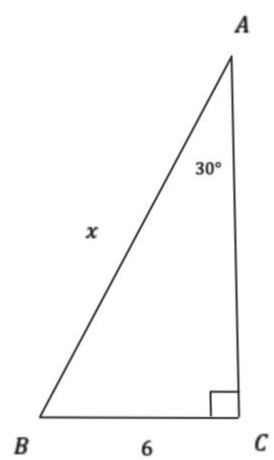

b.



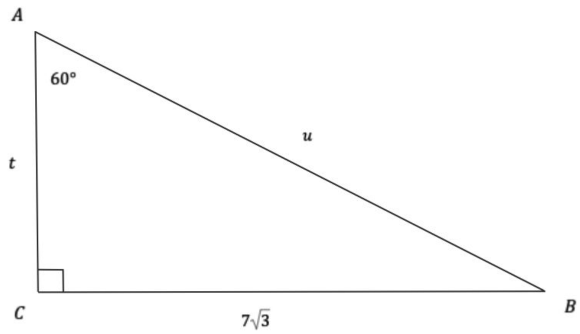
**Special Right Triangles**

- There are two special right triangles we need to know, and the SAT includes their ratios in its formula sheet:
  - 30º-60º-90º triangles are important because every equilateral triangle is made up of two of these special triangles.
  - 45º-45º-90 triangles are important because they are isosceles right triangles AND their hypotenuse represents the diagonal of a square.

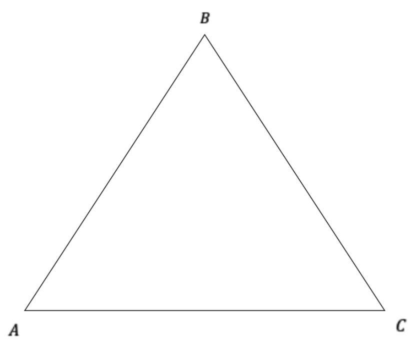
*55. Answer the following questions:*



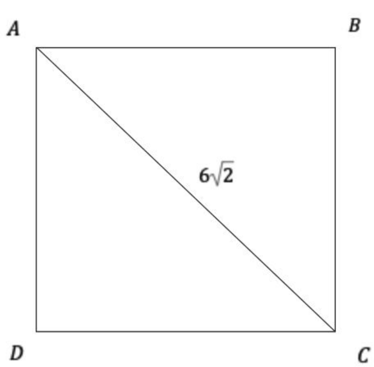

a. For the right triangle shown above, what is the value of $x$?




b. For the right triangle shown above, what is the value of $u + t$?




c. The perimeter of the equilateral triangle shown above is 24. What is the area of the triangle?

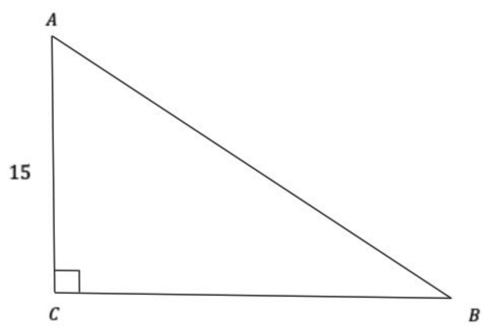



d. Given the diagram of square $ABCD$ shown above, what is the area of the square?

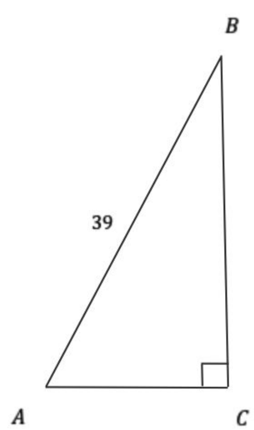
## SOHCAHTOA

- SOHCAHTOA is a helpful mnemonic to remember the three primary trigonometric functions:
  - SOH tells us that $sin(\theta) = \frac{Opposite}{Hypotenuse}$
  - CAH tells us that $cos(\theta) = \frac{Adjacent}{Hypotenuse}$
  - TOA tells us that $tan(\theta) = \frac{Opposite}{Adjacent}$
- Trig ratios may be reduced, so a sine of ⅘ does not mean that the hypotenuse must be 5; it simply means that the ratio of the opposite leg to the hypotenuse can be reduced to ⅘.

*56. Use SOHCAHTOA to solve the following questions:*




a. In the right triangle shown above, $sin(B) = \frac{3}{4}$. What is the length of side $AB$?

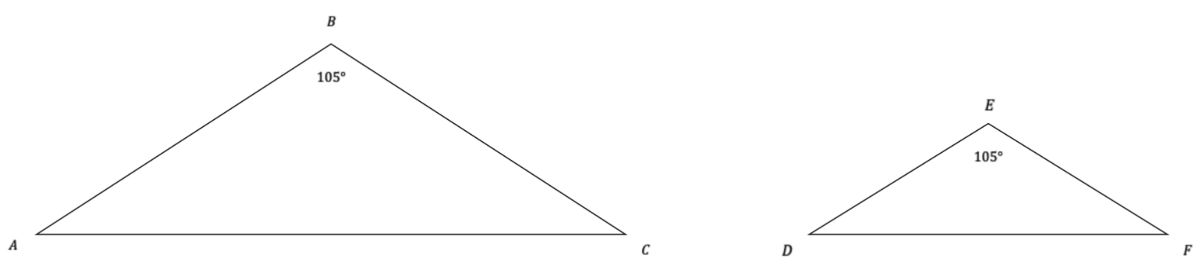



b. In the right triangle shown above, $tan(B) = \frac{5}{12}$. What is the length of side $AC$?

**Similar Triangles**

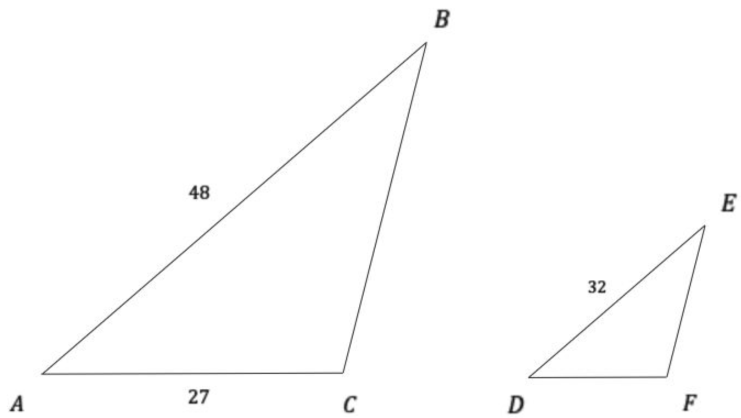
- Similar triangles have the same angle measures as one another, and their sides are in a consistent ratio.
- We can prove that two triangles are similar by proving that:
    - Two of the angles in each triangle are equal (then the third angles must be equal): AA
    - One angle in each triangle is equal AND the adjacent sides are in a consistent ratio: SAS
    - A line parallel to one side is drawn inside the triangle.
- When asked for side lengths of similar triangles, we can use a ratio to find the unknown value.

*57. Answer the following questions about similar triangles*:

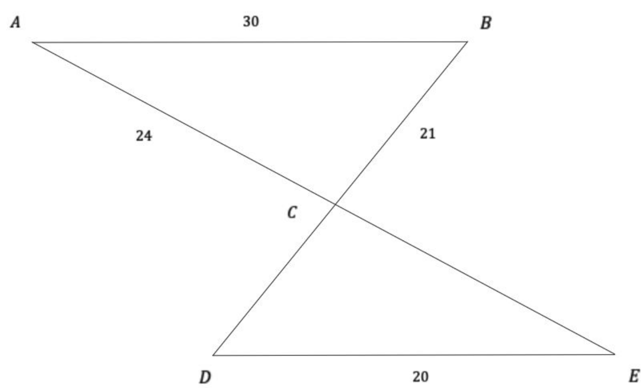



a. Given $\triangle ABC$ and $\triangle DEF$ shown above, what additional information, by itself, would be sufficient to prove that the two triangles are similar (circle all that apply):

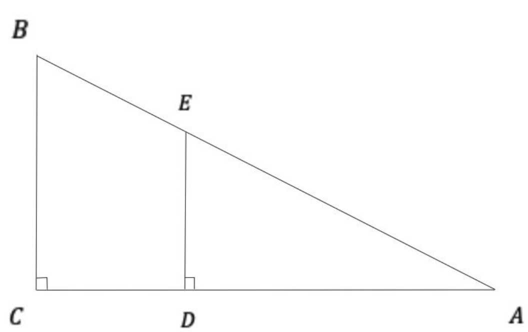
A. Angle $A$ is congruent to angle $B$
B. The length of $EF$ is $\frac{1}{2}$ the length of $BC$
C. Both triangles are isosceles triangles
D. Angle $A$ is congruent to angle $F$
E. Side $AC$ is congruent to side $DF$




b. Given that $\triangle ABC$ and $\triangle DEF$ shown above are similar, what is the length of $DF$?



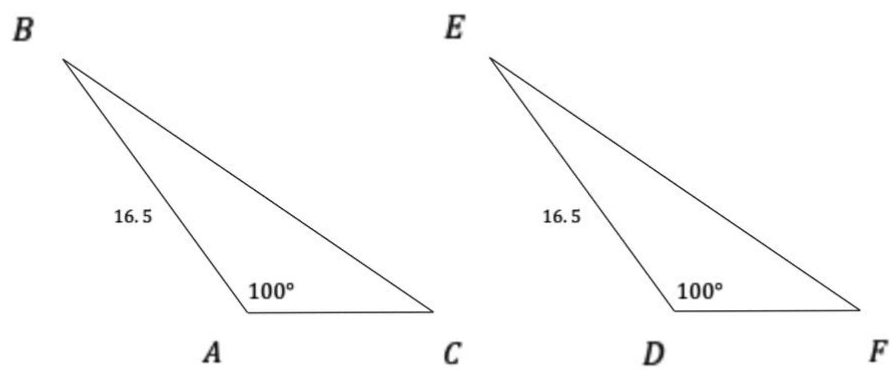

c. In the figure above, line segment $AB$ is parallel to line segment $DE$. What is the length of $CE$?




d. In the figure above, the length of $AB$ is 35, the $sin(A) = \frac{3}{5}$, and the length of $DC$ is 12. What is the length of $ED$?

- Congruent triangles are identical to one another: they have the same angle measures and their sides are equal in length.
- We can prove that two triangles are congruent by proving that:
    - All three corresponding sides are equal: SSS
    - Two corresponding sides and the angle between them are equal: SAS
    - Two corresponding angles and the side between them are equal: ASA
    - Two corresponding angles and one other side are equal: AAS

*58. Answer the following question about congruent triangles*:




a. Given $\triangle ABC$ and $\triangle DEF$ shown above, what additional information, by itself, would be sufficient to prove that the two triangles are congruent (circle all that apply):

A. Angle $C$ is congruent to angle $F$
B. Angle $B$ is congruent to angle $F$
C. Angle $B$ is congruent to angle $E$
D. Side $AC$ is congruent to side $DF$
E. Side $BC$ is congruent to side $EF$
F. Side $AC$ is congruent to side $EF$

**Quadrilaterals**

- We need to be prepared to find the area and perimeter of different types of quadrilaterals:
    - Squares: the area of a square is $s^2$ and the perimeter is $4s$.
    - Rectangles: the area of a rectangle is $l \bullet w$ and the perimeter is $2l + 2w$
    - Trapezoids: the area of a trapezoid is $h(\frac{(b_1+b_2)}{2})$
- The diagonal of a square or rectangle can be found using the Pythagorean theorem.
- Some questions ask us to compare the areas of shapes with different side lengths. If we know the ratios of side lengths, we need to square this ratio to find the ratio of areas.

*59. Answer the following questions about quadrilaterals*:

a. The perimeter of a square is 36. What is the area of the square?

b. The area of a rectangle is 48. If the length is two units greater than the width, what is the perimeter of the rectangle?

c. The length of a rectangle is 12 and the diagonal of the rectangle is 13. What is the area of the rectangle?

d. The diagonal of a square is $7\sqrt{2}$. What is the perimeter of the square?

e. Rectangle A has side lengths of 3 and 5. Rectangle B is constructed by doubling the length of each side of rectangle A. The area of rectangle B is how many times the area of rectangle A?

**Circles**

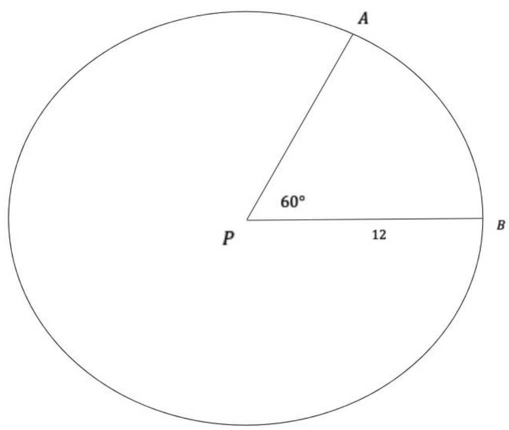
- The SAT will provide us with the formulas for finding the circumference and area of a circle, and we should reference these formulas whenever we are asked about circles.
- If we know one measure of a circle – its radius, diameter, circumference, or area – then we have enough information to find all other measures.

*60. Use the area and circumference formulas to complete the table*:

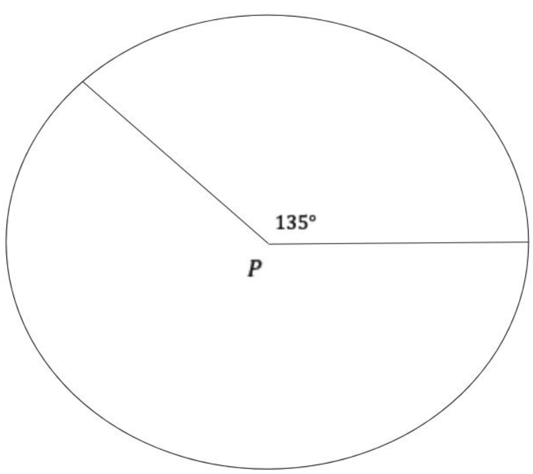
| Radius | Diameter | Circumference | Area |
|---|---|---|---|
| 4 | | | |
| | | $14\pi$ | |
| | $10$ | | |
| | | | $36\pi$ |
| | $1$ | | |

- An arc of a circle is a fraction of its circumference. The area of a sector is a fraction of the area of the circle. The central angle divided by 360º tells us what that fraction is.

*61. Answer the following questions about arcs and sectors:*




a. In the diagram of circle $P$ above, the measure of the central angle is $60°$ and the radius is $12$. What is the length of minor arc $AB$?




b. In the diagram above, circle $P$ has a circumference of $16\pi$. What is the area of the sector of the circle with a central angle of $135°$?

### 3D Shapes

- The formula page includes essential volumes for rectangular prisms, cylinders, cones, pyramids, and spheres.
- The surface area of a square is $6s^2$.
- The surface area of a rectangular prism is $2lw + 2lh + 2wh$.
- To find the density of a 3-dimensional solid we calculate the mass divided by the volume, resulting in answers such as grams/$cm^3$ or Kg/$m^3$.

*62. Answer the following questions*:

a. A rectangular box has a width of 5 feet, a length of 2 feet, and a height of 3 feet. What is the surface area of the box in square feet?

b. A cube has a surface area of 150 $cm^2$. What is the volume of the cube in $cm^3$?

c. Cylinder A has a volume of 24 $m^3$. Cylinder B's radius is twice as long as that of cylinder A and its height is half that of cylinder A. What is the volume of cylinder B?

d. The density of a sample of iron ore is 3 g/$cm^3$, and the density of a sample of magnetite is 5.2 g/$cm^3$. How much greater is the mass of a cube of magnetite with side lengths of 3 cm than the mass of a cube of iron ore of the same dimensions?

e. A cube with side lengths of ½ centimeters has a mass of 8 grams. What is the density of this cube in g/$cm^3$?

**Circles in the XY Plane**

The essential formula of a circle is:

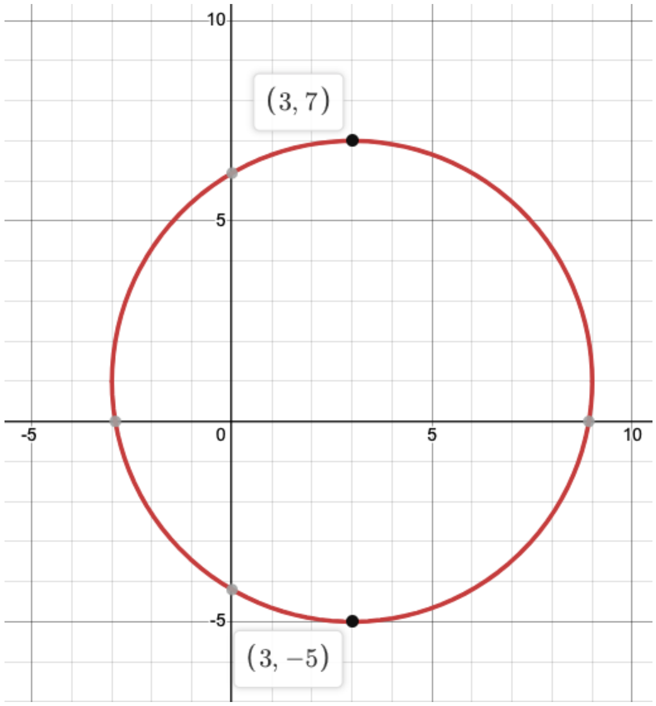
- $(x - h)^2 + (y - k)^2 = r^2$
    - $(h, k)$ is the center of the circle.
    - $r$ is the radius of the circle.
    - This formula is based on the Pythagorean theorem: $a^2 + b^2 = c^2$.
- When the equation is not in this form, we will need to complete the square for the $x$ terms and the $y$ terms.
    - If the equation looks like this: $ax^2 + bx + cy^2 + dy = e$, add $(\frac{b}{2})^2$ to both sides to create a perfect square for the $x$ terms, then repeat the process for $d$ to create a perfect square for the $y$ terms.
    - When we add $(\frac{b}{2})^2$ and $(\frac{d}{2})^2$, we must be sure to add it to BOTH sides of the equation.

*63. Answer the following questions*:

a. The equation of a circle is $(x + 2)^2 + (y - 3)^2 = 36$. What is the $y$ coordinate of the center of the circle?

b. The coordinate (2a,b) is the center of the circle $(x - 6)^2 + (y - 8)^2 = 25$. What is the value of $a$?

c. What is the area of a circle with the equation $(x + 1)^2 + y^2 = 49$?




d. What is the equation of the circle shown above?

e. A circle in the xy plane can be represented with the equation $x^2 + 6x + y^2 - 2y = 15$. What is the circumference of this circle?

f. Find the area of a circle in the xy plane that is represented by the equation $2x^2 - 16x + 2y^2 + 20y - 11 = 5$.

**Math Guide Answer Key**

1. Solving equations
   a. $x = 4$
   b. $5x =- 40$
   c. $10 - n = 6$
   d. $- a = 0$
   e. $x^2 = 25$
   f. $15x = 42$
2. Creating equations
   a. $n - 4 = 17$
   b. $18 + 3n = 46$
   c. $n = 62 - 15$
   d. $3n = 8 + (8n - 30)$
   e. $15n + 7 = 62$
   f. $45 + 0.07p = 132.50$
3. Number of solutions
   a. no solutions
   b. 1 solution
   c. infinite solutions
   d. no solutions
4. Inequalities
   a. $x < \frac{7}{4}$
   b. $x > \frac{21}{8}$
   c. $x \leq \frac{3}{11}$
   d. $x \geq \frac{43}{6}$
5. Absolute value
   a. $x = 3, \ - 7$
   b. $x = 8, \ - 2$
   c. no solution
   d. $x = 11, \ - 1$
6. Algebraic rearranging
   a. $x = \frac{y-4}{2}$
   b. $b = \frac{4a+27}{2}$
   c. $n = \frac{b+c-3f}{6}$
   d. $c = \frac{4a+4d}{3a-2}$
7. Creating linear expressions
   a. $y =- 5x + 7$
   b. $y =- x + 4$
   c. $y = 3x - 25$
   d. $y = \frac{2}{3}x + 3$
   e. $y =- 4x + 13$
   f. $y = 4x - 8$
   g. $c = 50d + 35$
   h. $y = 40x + 30$
8. Solving linear expressions
   a. $32.50
   b. 150 grams
   c. 9 dogs
   d. 21 seconds
9. Interpreting linear expressions
   a. Tilda starts the test with 125 minutes
   b. The social media manager is paid $45 per hour for editing videos
   c. If the factory produces 15 square tiles in a day, it can also produce 30 rectangular tiles

10. Systems of linear equations

a. zero
b. infinity
c. one
d. one
e. zero
f. zero
g. infinity
h. one
i. zero

11. Solving systems of equations (note: answers may vary. Graphing will always work)

a. substitution
b. elimination
c. substitution

12.

a. $x + y = -8$
b. $2x + 2y = 40$

13. Creating systems of equations

a. $4s + 9h = 38$ ; $s + h = 7$; $s = 5$
b. $2w + 3h = 34$ ; $w + h = 15$; $h = 4$
c. $14g + 12n = 132$ ; $g = n + 2$; N earned $48

14. Creating systems of inequalities

a. $20h + 15m \geq 400$ ; $h + m \leq 25$
b. $t + z \geq 15$ ; $50t + 20z \leq 500$

15. $m + f \leq 20$ ; $5m + 7f \geq 110$

16. Solving systems of inequalities

a. d, e
b. j

17. Identifying quadratic expressions

b, c, f, g, h, j, l

18. Properties of parabolas

a. upwards: b, c, f, g, h
b. downwards: j, l

19. FOILing quadratic expressions

a. $y = x^2 + 2x - 24$
b. $y = 3x^2 - x - 2$
c. $y = x^2 - 10x + 21$

20. Factoring quadratic expressions

a. $y = (x + 3)(x + 2)$
b. $y = (x - 4)(x + 1)$
c. $y = (x + 14)(x - 2)$

21.

a. $x = -5, -6$
b. $x = 5$
c. $x = -9, 2$

22.

a. $x = -1.5, -4$
b. $x = \frac{1}{3}, 5$
c. $x = -1$

23.

a. $x = 0, 1$
b. $x = -9, 2$
c. $x = \frac{4}{3}, -1$

24. Difference of perfect squares

a. $y = x^2 - 9$
b. $y = x^2 - 25$
c. $y = x^2 - 100$

25.

a. $y = (x + 4)(x - 4)$
b. $y = (x + 7)(x - 7)$
c. $y = (x + 6)(x - 6)$

26. Matching corresponding terms
   a. $a = 3,\ b = 16,\ c =- 9$
   b. $a = 1,\ b =- 11,\ c = 53$
   c. $a = 5,\ b = (2 - u),\ c = 36$
   d. $a =- 1,\ b = k,\ c = 0$
   e. $a = 1,\ b =- 6,\ c =- 49$
   f. $a = 2,\ b = 4,\ c =- 7 + t$

27.
   a. $k = 10$
   b. $h + v =- 13$
   c. $n = 20$

28. Finding the vertex
   a. $(2,\ 17)$
   b. $(1,\ \ - 16)$
   c. $(0, - 36)$

29. Forms of quadratic expressions
   a. standard
   b. vertex
   c. factored
   d. vertex
   e. standard
   f. none

30. Systems of equations w/ Quadratic
   a. $x =- 3,\ 2$
   b. $y =- 4,\ 8$
   c. $y = 30$

31. The quadratic formula
   a. $x =- 3 \pm 2\sqrt{3}$
   b. $x = 5 \pm \sqrt{2}$
   c. $x =- 1 \pm \sqrt{6}$

32.
   a. $x =- 2$
   b. no solution
   c. one solution
   d. $f(x) = 21$
   e. no solution
   f. $b =- 6\ or\ 6$

33. Sum of solutions
   a. $- 3$
   b. $\frac{8}{7}$
   c. $- 11$
   d. $5$

34. Functions
   a. $f(3) =- 1$
   b. $g(9) = 87$
   c. $f(22) = 37$
   d. $n =\pm 5$
   e. $f(f(4)) =- 5$
   f. $g(f(8)) = 87$
   g. $k = 9$
   h. $m =\pm 2$
   i. $v = 6$
   j. $p =\pm 7$

35. f(x) tables
   a. $f(1) = 3$
   b. $g(- 2) = 10$
   c. $f(0) = 5$
   d. $m = 0$
   e. $g(f(- 1)) = 10$
   f. $n =- 1$
   g. $k = 3$
   h. $h(1) =- 6$

36. f(x) graphs

a. $f(4) = 1$
b. $g(-2) = 0$
c. $p = 1$
d. $f(g(3)) = -1$
e. $n = 2, \ -2$
f. $n = -3, -2, 0$
g. $f(3) + g(3) = -2$
h. $-3 \leq x \leq 0$

37. Exponent operations

a. $x^{27}$
b. $\frac{4}{\sqrt{x}}$
c. $32w^3$
d. $x^3y^4$
e. $12x^6 + 4x^4$
f. $\frac{c^{5/3}\sqrt{b}}{a^8}$

38. Radical operations

a. $8\sqrt{2}$
b. $3$
c. $2\sqrt[3]{4}$
d. $3\sqrt{3}$
e. $20\sqrt{3}$
f. $20\sqrt{2}$
g. $x^2$
h. $30x^5$
i. $7xy^2\sqrt{2xy}$

39.

a. $x = 2; \ -5 \, is \, extraneous$
b. $x = 10; \ 5 \, is \, extraneous$
c. $x = 7; \ -3 \, is \, extraneous$

40. Exponential growth graphs

a. 4
b. 2
c. 1
d. 3

41.

a. 2
b. 1
c. 4
d. 3

42. Percents

a. $0.19n = 38; \ n = 200$
b. $0.14 \cdot 50 = n; \ n = 21$
c. $220p = 44; \ p = 0.2 \, or \, 20\%$
d. $0.3n = 75; \ n = 250$
e. $1.18 \cdot 50 = n; \ n = 59$

43. Percent change

a. $25 \cdot 1.2 = 30$
b. $40 \cdot 1.05 = 42$
c. $220 \cdot 0.7 = 154$
d. $76 \cdot 2.5 = 190$
e. $50 \cdot 1.1 = 55; \ 55 \cdot 0.8 = 44$
f. $350 \cdot 0.4 = 140; \ 140 \cdot 1.4 = 196$
g. $140 \cdot 1.15 = 161; 161 \cdot 1.05 = 169.05$
h. $200 \cdot 0.7 = 140; \ 140 \cdot 2.1 = 294$

44.

a. $1.4n = 280; \ n = 200$
b. $(1.25 \cdot n) \cdot 0.5 = 75; \ n = 120$
c. $(250 \cdot 1.12)p = 224; \ p = 20\%$

45. Exponential growth and decay

   a. $y = 1,500(1.04)^{x}$

   b. $y = 650,000(1.1)^{\frac{x}{8}}$

   c. $y = 40,000(0.95)^{\frac{q}{8}}$

   d. $y = 140(\frac{1}{2})^{\frac{x}{245}}$

46. Mean, median, mode, range

   a. mean= 8.57
      median= 5
      mode= 5
      range= 18

   b. mean= 13.8
      median= 19
      mode= none
      range= 22

   c. mean= -46.25
      median= -45
      mode= -90 and -45
      range= 90

47.

   a. mean= 70.32
      median= 71
      mode= 72
      range= 4

   b. mean= 1.85
      median= 1.5
      mode= 1
      range= 5

   c. mean= 2.6
      median= 2
      mode= 1 and 2
      range= 6

   d. mean= 4
      median= 4
      mode= 4
      range= 6

   e. 10 ≤ median < 15

48. Standard deviation

   a. Set B
   b. Set A
   c. Set A
   d. Equal

49. Proportions

   a. 19.5 hours
   b. 4.5 envelopes/minute
   c. 78 students
   d. 15 fewer minutes

50. Probability
    a. $\frac{71}{172}$ $or$ $\approx 0.413$
    b. 243
    c. 0.0835
    d. 0.267
51. Margin of error
    a. [50.2, 57.2]
    b. Our range of plausible values for the mean is [97, 151], so 172g is unlikely to be the true mean mass of carrots grown on this farm since it is outside of that interval.
52. Angles
    a. c/e, d/f
    b. b/d, a/c, e/g, h/f
    c. c/f, d/e
    d. a/e, b/f, d/h, c/g
    e. b/g, a/h
    f. a/b, b/c, c/d, d/a, e/f, f/g, g/h, or h/e
    g. a/g, b/h
53.
    a. 44°
    b. a-b=146°
    c. x= 31
    d. x=75°
54. Triangles
    a. x= 24
    b. x= 8
55. Special right triangles
    a. x= 12
    b. u+t = 21
    c. $16\sqrt{3}$
    d. 36
56. SOHCAHTOA
    a. 20
    b. 15
57. Similar triangles
    a. C, D
    b. 18
    c. 16
    d. 12
58. Congruent triangles
    a. A, C, D
59. Quadrilaterals
    a. A = 81
    b. P = 28
    c. A = 60
    d. P = 28
    e. $A_B$ is 4 times greater than $A_A$
60. Circles
    a. Row 1: *r=4*, d=8, C=8π, A=16π
    b. Row 2: r=7, d=14, *C=14π*, A=49π
    c. Row 3: r=5, *d=10*, C=10π, A=25π
    d. Row 4: r=6, d=12, C=12π, *A=36π*
    e. Row 5: r=0.5, *d=1*, C=π, A=0.25π
61.
    a. 4π
    b. 24π

62. 3D shapes
    a. 62 $ft^2$
    b. 125 $cm^3$
    c. 48 $m^3$
    d. 59.4 g
    e. 64 $g/cm^3$

63. Circles in the XY plane
    a. y = 3
    b. a = 3
    c. A = 49π
    d. $(x - 3)^2 + (y - 1)^2 = 36$
    e. C = 10π
    f. A = 49π

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